$\underset{\sim}{1} = \begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix}$ $X = \begin{bmatrix}1&0&0\\1&0&0\\1&0&0\\0&1&0\\0&1&0\\0&1&0\\0&0&1\\0&0&1\\0&0&1\end{bmatrix}$

The SAS System

The GLM Procedure

Dependent Variable: y

$y'(P_X - P_{\underset{\sim}{1}})y$ $\quad y'(I-P_X)y$ $\quad y'(P_X - P_{\underset{\sim}{1}})y/2$ $\quad y'(I-P_X)y/6$

$\dfrac{y'(P_X-P_{\underset{\sim}{1}})y/2}{y'(I-P_X)y/6}$

Source		DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	RANK(X) - RANK($\underset{\sim}{1}$)	2	214.2222222	107.1111111	18.54	0.0027
Error	n - RANK(X)	6	34.6666667	5.7777778		
Corrected Total	n - 1	8	248.8888889			

$P(F_{2,6} \geq 18.54)$

$y'(I-P_{\underset{\sim}{1}})y$

R-Square	Coeff Var	Root MSE	y Mean
0.860714	13.43684	2.403701	17.88889

$\sqrt{MSE}$ $\quad = \bar{y}_{\cdot\cdot}$

$\dfrac{214.\bar{2}}{248.\bar{8}}$ $\quad \dfrac{100\sqrt{MSE}}{\bar{y}_{\cdot\cdot}}$

Source	DF	Type I SS	Mean Square	F Value	Pr > F
x	2	214.2222222	107.1111111	18.54	0.0027

Source	DF	Type III SS	Mean Square	F Value	Pr > F
x	2	214.2222222	107.1111111	18.54	0.0027

$$X_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 2 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 3 \end{bmatrix}$$

$$3 - 2 = 1$$

$$\text{RANK}(X) - \text{RANK}(X_0)$$

$$y'(P_X - P_{X_0})y$$

$$\frac{y'(P_X - P_{X_0})y}{\text{RANK}(X) - \text{RANK}(X_0)}$$

$$P(F_{1,6} \geq 7.54)$$

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Lack of Linear Fit	1	43.55555556	43.55555556	7.54	0.0335

$$C = [1, -2, 1]$$

$$\hat{\beta} = \begin{bmatrix} \bar{y}_{1\cdot} \\ \bar{y}_{2\cdot} \\ \bar{y}_{3\cdot} \end{bmatrix}$$

$$(C\hat{\beta})'[C(X'X)^- C']^{-1} C\hat{\beta}$$

$$\left[(C\hat{\beta})'[C(X'X)^- C']^{-1} C\hat{\beta} / 1\right]$$

$$F = \frac{\left[\dfrac{y'(P_X - P_{X_0})y}{\text{RANK}(X) - \text{RANK}(X_0)}\right]}{\left[\dfrac{y'(I - P_X)y}{n - \text{RANK}(X)}\right]} = \frac{(C\hat{\beta})'[C(X'X)^- C']^{-1} C\hat{\beta}/1}{\hat{\sigma}^2}$$