

# SAS Code

```
proc mixed;  
  class diet drug;  
  model weightgain=diet*drug / noint;  
  lsmeans diet*drug;
```

$$\begin{aligned}
 Y &= \begin{bmatrix} Y_{111} \\ Y_{112} \\ Y_{121} \\ Y_{122} \\ \vdots \\ \vdots \\ \vdots \\ Y_{231} \\ Y_{232} \end{bmatrix} & X &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow X'X = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \Rightarrow X'X = 2 I_{6 \times 6} \\
 & & & & & \Rightarrow (X'X)^{-1} = (X'X)^{-1} = \frac{1}{2} I_{6 \times 6}
 \end{aligned}$$

$$\begin{aligned}
 X'Y &= \begin{bmatrix} Y_{11.} \\ Y_{12.} \\ Y_{13.} \\ Y_{21.} \\ Y_{22.} \\ Y_{23.} \end{bmatrix} \Rightarrow \hat{\beta} = (X'X)^{-1} X'Y = \frac{1}{2} I_{6 \times 6} \begin{bmatrix} Y_{11.} \\ Y_{12.} \\ Y_{13.} \\ Y_{21.} \\ Y_{22.} \\ Y_{23.} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11.} \\ \bar{Y}_{12.} \\ \bar{Y}_{13.} \\ \bar{Y}_{21.} \\ \bar{Y}_{22.} \\ \bar{Y}_{23.} \end{bmatrix} \\
 & & & & & \uparrow
 \end{aligned}$$

THESE ARE OLS ESTIMATES AND LS MEANS FOR EACH COMPONENT

$$\text{OF } \beta = \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \vdots \\ \mu_{23} \end{bmatrix}.$$

# SAS Output

$$\underline{c}'_{i1} = [1, 0, 0, 0, 0, 0]$$

$$\underline{c}'_{i2} = [0, 1, 0, 0, 0, 0]$$

$$\vdots$$

$$\underline{c}'_{i6} = [0, 0, 0, 0, 0, 1]$$

$$\underline{c}'_{ij} (X'X)^{-1} \underline{c}_{ij} = \underline{c}'_{ij} \left(\frac{1}{2} I\right) \underline{c}_{ij}$$

$$= \frac{1}{2} \underline{c}'_{ij} I \underline{c}_{ij}$$

$$= \frac{1}{2} \underline{c}'_{ij} \underline{c}_{ij}$$

$$= \frac{1}{2}$$

## Least Squares Means

Effect	diet	drug	Estimate	Standard Error	DF	t Value
	<i>i</i>	<i>j</i>	$\hat{\mu}_{ij} = \bar{y}_{ij} = \underline{c}'_{ij} \hat{\beta}$	$\sqrt{\hat{\sigma}^2 \underline{c}'_{ij} (X'X)^{-1} \underline{c}_{ij}} = \sqrt{\hat{\sigma}^2 / 2}$		
diet*drug	1	1	42.5000	0.7832	6	54.27
diet*drug	1	2	40.0500	0.7832	6	51.14
diet*drug	1	3	37.6500	0.7832	6	48.07
diet*drug	2	1	35.7000	0.7832	6	45.58
diet*drug	2	2	33.9500	0.7832	6	43.35
diet*drug	2	3	35.4500	0.7832	6	45.27

$$\frac{n-r}{12-6} \frac{\text{ESTIMATE}}{\text{SE(ESTIMATE)}} = 6$$