1a) A_1: $\frac{11+7+2}{3} = \frac{20}{3} = 6.\bar{6}$

A_2: $\frac{10+9+5}{3} = 8$

1b) EACH OF THE SAMPLE VARIANCES IS AN UNBIASED ESTIMATOR OF σ^2. THUS, WE USE A DF-WEIGHTED AVERAGE OF THESE MEAN SQUARES TO ESTIMATE σ^2.

$$(5-1)3 + (3-1)1 + (2-1)2$$
$$+ (3-1)2 + (5-1)3 + (5-1)1 = 36$$

$$\Longrightarrow \hat{\sigma}^2 = \frac{36}{5+3+2+3+5+5-6} = \frac{36}{17}$$

1c) $2-5 \pm t_{17,.975}\sqrt{\frac{36}{17}\left[\frac{1}{2}+\frac{1}{5}\right]}$

1d) $H_0: \frac{\mu_{11}+\mu_{12}+\mu_{13}}{3} = \frac{\mu_{21}+\mu_{22}+\mu_{23}}{3}$

$\Longleftrightarrow H_0: (\mu_{11}+\mu_{12}+\mu_{13})-(\mu_{21}+\mu_{22}+\mu_{23}) = 0$

$$t = \frac{(11+7+2)-(10+9+5)}{\sqrt{\frac{36}{17}\left(\frac{1}{5}+\frac{1}{3}+\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{5}\right)}}$$

1 e) IF B_1 INVOLVES APPLICATION OF 0 UNITS OF THE CHEMICAL TO SOIL, THEN TREATMENTS $A_1 B_1$ AND $A_2 B_1$ ARE IDENTICAL. THIS IMPLIES $\mu_{11} = \mu_{21}$. THUS, WE SHOULD HAVE FIVE TREATMENT MEANS INSTEAD OF SIX IN OUR MODEL.

2a) TWO MODELS ARE FIT TO THE DATA IN THE R CODE AND OUTPUT. BOTH MODELS ASSUME

$y_i \mid \lambda_i \sim \text{POISSON}(\lambda_i)$ WITH INDEPENDENCE OF $y_i \mid \lambda_i$ ACROSS $i = 1, \ldots, 75$.

IN MODEL 1,

$$\lambda_i = \exp\{\beta_1 + \beta_2 x_i\}$$

IN MODEL 2,

$$\lambda_i = \exp\{\beta_1 + \beta_2 x_i + \beta_3 1_{[s_i=2]} + \beta_4 1_{[s_i=3]} + \beta_5 x_i 1_{[s_i=2]} + \beta_6 x_i 1_{[s_i=3]}\}$$

WHERE $1_{[s_i=c]} = \begin{cases} 1 & \text{IF } s_i = c \\ 0 & \text{OTHERWISE.} \end{cases}$

2a) (CONTINUED)

THE QUESTION IS ASKING FOR A TEST OF

$$H_0: \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0.$$

WE COULD CONSIDER A LIKELIHOOD RATIO TEST OF REDUCED VS. FULL MODEL, WHERE MODEL 1 IS REDUCED AND MODEL 2 IS FULL. THAT STATISTIC WOULD BE

$$2\hat{\ell}_2 - 2\hat{\ell}_1 = (2\hat{\ell}_s - 2\hat{\ell}_1) - (2\hat{\ell}_s - 2\hat{\ell}_2)$$

$$= 1204.1 - 239.27$$

HOWEVER, IT LOOKS LIKE DATA ARE OVERDISPERSED RELATIVE TO THE POISSON DISTRIBUTION.

2a) (CONTINUED)

TO SEE EVIDENCE OF OVERDISPERSION, NOTE THAT

$$2\hat{\ell}_S - 2\hat{\ell}_2 = 239.27.$$

UNDER THE NULL THAT SAYS THE FIT OF MODEL 2 IS ADEQUATE, THIS STATISTIC IS χ^2 WITH 69 DF.

THE SD OF χ^2_{69} IS $\sqrt{(2)(69)} < 13$

THE MEAN OF χ^2_{69} IS 69.

THUS, 239.27 IS MANY STANDARD DEVIATIONS ABOVE AVERAGE. WE CAN ADJUST FOR OVERDISPERSION WITH A QUASI LIKELIHOOD APPROACH.

$\hat{\phi} = 239.27/69.$ $\quad F = \dfrac{(1204.1 - 239.27)/4}{239.27/69}$

2b) F WITH 4 AND 69 DF

2c) $\exp(0.438769 + 0.595104 + (0.030749 + 0.013336)x)$

3a) THIS IS A SPLIT-PLOT EXPERIMENT. THE WHOLE-PLOT PART OF THE EXPERIMENT IS ARRANGED AS A CRD WITH NO BLOCKING.

THE TWO SECTIONS TAUGHT BY ANY ONE INSTRUCTOR COULD BE CONSIDERED A BLOCK OF TWO SPLIT-PLOT EXPERIMENTAL UNITS TWO WHICH THE LEVELS OF TESTING METHOD ARE RANDOMLY ASSIGNED.

3b) UNIVERSITIES ARE WHOLE-PLOT EXPERIMENTAL UNITS.

SECTIONS ARE SPLIT-PLOT EXPERIMENTAL UNITS.

THE DETERMINATION OF THE EXPERIMENTAL UNITS FOLLOWS FROM THE RANDOM ASSIGNMENT OF UNIVERSITIES TO TEACHING STYLES AND SECTIONS TO TESTING METHODS.

3c)

Source	DF
TeachStyle	1
Univ(TeachStyle)	8
Inst(Univ,TeachStyle)	10
TestMethod	1
TeachStyle x TestMethod	1
Section $\left\{ \begin{array}{l} \text{TestMethod x Univ(TeachStyle) +} \\ \text{TestMethod x Inst(Univ, TeachStyle)} \end{array} \right.$	18
Student(TestMethod, Inst, Univ, TeachStyle)	1160
C. Total	1199

3d) THE 18 DF TERM ABOVE, WHICH COULD BE CALLED "SECTION".

3e) Univ(TeachStyle)

4 a)

Null	Alternative	LRTS	DF
C	A	$2(191.7 - 189.9)$	5
D	B	$2(194.5 - 192.4)$	5

In no other pair is one model a special case of the other.

b) Both of the test stats in the table above are less than the null expectation (DF = 5). Thus, p-values will be large and null models favored.

Between C & D, both models have the same number of parameters, and C has the higher likelihood. Hence C will have lower AIC & BIC and would be preferred over D.

4c) $$\hat{\underset{\sim}{\mu}}_1 = \begin{bmatrix} \hat{\mu}_{10} \\ \hat{\mu}_{11} \\ \hat{\mu}_{12} \\ \hat{\mu}_{13} \end{bmatrix} = \begin{bmatrix} 16 \\ 16+8 \\ 16+14 \\ 16+25 \end{bmatrix} = \begin{bmatrix} 16 \\ 24 \\ 30 \\ 41 \end{bmatrix}$$

$$\hat{\underset{\sim}{\mu}}_2 = \begin{bmatrix} \hat{\mu}_{20} \\ \hat{\mu}_{21} \\ \hat{\mu}_{22} \\ \hat{\mu}_{23} \end{bmatrix} = \begin{bmatrix} 16-4 \\ 16-4+8-8 \\ 16-4+14-12 \\ 16-4+25-19 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 14 \\ 18 \end{bmatrix}$$

5a) $\log\left(\frac{\hat{\pi}_i}{1-\hat{\pi}_i}\right) = \hat{\theta} \iff \frac{1}{1+\exp(-\hat{\theta})} = \frac{1}{1+\exp(0.9)}$

b) $$\frac{\hat{\theta} - \theta - e_{51}}{\sqrt{SE(\hat{\theta})^2 + \hat{\sigma}^2}} \dot{\sim} N(0,1)$$

$$\Rightarrow \Pr\left[-0.9 - 2\sqrt{0.1^2 + 0.05} \le \theta + e_{51} \le -0.9 + 2\sqrt{0.1^2 + 0.05}\right] \approx .95$$

$\Rightarrow -0.9 \pm 2\sqrt{0.06}$ APPROXIMATE 95% INTERVAL FOR $\theta + e_{51}$

$$\Rightarrow \frac{1}{1+\exp(0.9 + 2\sqrt{0.06})} \text{ TO } \frac{1}{1+\exp(0.9 - 2\sqrt{0.06})}$$

IS APPROXIMATE 95% PREDICTION INTERVAL FOR π_{51}.