OVERDISPERSION SUMMARY

$\hat{\ell}_S$ = MAXIMIZED SATURATED LOG LIKELIHOOD

$\hat{\ell}_F$ = MAXIMIZED LOG LIKELIHOOD FOR FULL MODEL

$\hat{\ell}_R$ = MAXIMIZED LOG LIKELIHOOD FOR REDUCED MODEL

$\hat{\ell}_O$ = MAXIMIZED LOG LIKELIHOOD FOR INTERCEPT-ONLY MODEL

MODEL	DIMENSION OF MODEL PARAMETER SPACE
S	n
F	p
R	$p-q$
O	1

DIFFERENCE $= q$

TO CHECK FOR OVERDISPERSION IN FULL MODEL:

COMPARE DEVIANCE STATISTIC $= 2\hat{\ell}_S - 2\hat{\ell}_F$

$$= \sum_{i=1}^{n} d_i^2$$

OR THE PEARSON STATISTIC $= \sum_{i=1}^{n} r_i^2$ TO A χ^2_{n-p} DISTRIBUTION. IF THE LACK-OF-FIT STATISTIC $\sum_{i=1}^{n} d_i^2$ OR $\sum_{i=1}^{n} r_i^2$ IS UNUSUALLY LARGE FOR THE χ^2_{n-p} DISTRIBUTION, CONSIDER ADJUSTING FOR OVERDISPERSION.

To Adjust For Overdispersion, Compute

$$\hat{\phi} = \frac{\text{Lack-of-Fit Statistic}}{n-p},$$

Where Lack-of-Fit Statistic Can Be Either The

$$\text{Deviance Statistic} = \sum_{i=1}^{n} d_i^2$$

or The

$$\text{Pearson Statistic} = \sum_{i=1}^{n} r_i^2.$$

Then Perform Inference As Follows.

Approximate $100(1-\alpha)\%$ Confidence Interval For $\underline{c}'\underline{\beta}$:

$$\underline{c}'\hat{\underline{\beta}} \pm t_{1-\alpha/2,\,n-p}\sqrt{\hat{\phi}\,\underline{c}'\hat{I}^{-1}(\hat{\underline{\beta}})\,\underline{c}}$$

Test of $H_0: \underline{c}'\underline{\beta} = d$ Based On

$$\frac{\underline{c}'\hat{\underline{\beta}} - d}{\sqrt{\hat{\phi}\,\underline{c}'\hat{I}^{-1}(\hat{\underline{\beta}})\,\underline{c}}} \,\dot{\sim}\, t_{n-p} \text{ Under } H_0$$

TEST OF $H_0: C\underline{\beta} = \underline{d}$ BASED ON

$$\frac{(C\hat{\underline{\beta}} - \underline{d})' \left[C \hat{\underline{I}}^{-1}(\hat{\underline{\beta}}) C' \right]^{-1} (C\hat{\underline{\beta}} - \underline{d}) / q}{\hat{\phi}}$$

$\dot{\sim} F_{q,\, n-p}$ UNDER H_0, WHERE

C IS A $q \times p$ MATRIX OF RANK q.

TEST OF H_0: "REDUCED MODEL R IS ADEQUATE COMPARED TO FULL MODEL F" IS BASED ON

$$\frac{(2\hat{\ell}_F - 2\hat{\ell}_R)/q}{\hat{\phi}} \;\dot{\sim}\; F_{q,\, n-p} \text{ UNDER } H_0.$$

A SPECIAL CASE OF THE REDUCED VS. FULL MODEL TEST IS WHEN THE REDUCED MODEL IS THE INTERCEPT-ONLY MODEL. THIS TEST IS BASED ON

$$\frac{\left(2\hat{l}_F - 2\hat{l}_0\right)/(p-1)}{\hat{\phi}} \;\dot{\sim}\; F_{p-1,\,n-p}$$

UNDER H_0: "THE INTERCEPT ONLY MODEL IS ADEQUATE RELATIVE TO THE FULL MODEL."

THIS STATISTIC COULD BE COMPUTED AS

$$\frac{(\text{NULL DEVIANCE} - \text{RESIDUAL DEVIANCE})/(p-1)}{\text{RESIDUAL DEVIANCE}/(n-p)}$$