

Instructions: This is a closed-notes, closed-book exam. No calculator or electronic device of any kind may be used. Use nothing but a pen or pencil. Please write your name and answers on blank paper. Please do NOT write your answers on the pages with the questions. For questions that require extensive numerical calculations that you should not be expected to do without a calculator, simply set up the calculation and leave it at that. For example, $(3.45 - 1.67)/\sqrt{2.34}$ would be an acceptable answer. On the other hand, some quantities that are very difficult to compute one way may be relatively easy to compute another way. Part of this exam tests your ability to figure out the easiest way to compute things, based on the information provided and the relationships between various quantities. If you find yourself trying to do exceedingly complex or tedious calculations, there is probably a better way to solve the problem.

1. Suppose \mathbf{y} is an $n \times 1$ random vector with distribution $N(\mathbf{0}, \Sigma)$, where Σ is a positive definite variance matrix with orthonormal eigenvectors $\mathbf{p}_1, \dots, \mathbf{p}_n$ and corresponding eigenvalues $\lambda_1, \dots, \lambda_n$. Find the distribution of the random vector $[\mathbf{p}'_1 \mathbf{y}, \mathbf{p}'_2 \mathbf{y}, \dots, \mathbf{p}'_n \mathbf{y}]'$ and simplify your answer as much as possible.

2. A meat scientist is studying the effect of storage temperatures and preservatives on meat quality. The temperatures of interest are 34, 40, and 46 degrees Fahrenheit. The two preservatives considered in the experiment are labeled $P1$ and $P2$. Twelve coolers are available for the experiment. The three temperatures are randomly assigned to the twelve coolers using a balanced and completely randomized design. Two large cuts of fresh beef are stored in each cooler. Within each cooler, one of the two cuts of beef is randomly selected and treated with preservative $P1$ while the other is treated with preservative $P2$. After three days, each member of a team of experts independently assigns a quality score to each cut of beef. The experts are not told about the storage conditions or preservative applied to each cut. The scores assigned by the team to each cut of beef are averaged to produce an overall quality score for each cut.
 - (a) What are the experimental units in this experiment?
 - (b) Provide the *Source* and *Degrees of Freedom* columns of the ANOVA table that corresponds to the model you would fit to the overall quality score data. You are not required to write down your model; you are only required to provide the entries in the *Source* and *Degrees of Freedom* columns of the ANOVA table corresponding to your model.
 - (c) Write the name of the term in the *Source* column whose mean square would be the denominator of the F -statistic for testing for temperature main effects.
 - (d) Write the name of the term in the *Source* column whose mean square would be the denominator of the F -statistic for testing for the main effect of preservative.

3. Scientists were interested in studying the kernel nutritional content of two varieties of maize (i.e., corn) treated with with different soil and kernel treatments. Eight plots of soil, arranged in four blocks of two plots each, were used in the initial stage of an experiment. Two soil treatments ($S1$ and $S2$) were randomly assigned to the plots using a randomized complete block design. Two rows of maize plants were planted in each plot. In each plot, one row was randomly chosen to be plants of variety $V1$ while the other row consisted of plants of variety $V2$. At harvest time, two plants were randomly selected from each row. One ear of corn was obtained from each of the selected plants. Two kernel treatments ($K1$ and $K2$) were randomly assigned to the two ears that were selected from each row. Then ten kernels of corn were randomly sampled from each ear and treated with the assigned kernel treatment. This process produced a total of 320 treated kernels that were stored for six months. After storage, the nutritional content of each of the 320 kernels was obtained by chemical analysis. Suppose the value of one response variable measuring the overall nutritional value was computed for each kernel for comparison of soil treatments, varieties, and kernel treatments. Provide the *Source* and *Degrees of Freedom* columns of the ANOVA table that corresponds to the model you would fit to the 320 measurements of overall nutritional value. You are not required to write down your model; you are only required to provide the entries in the *Source* and *Degrees of Freedom* columns of the ANOVA table corresponding to your model.

4. Scientists were interested in how shaving the head of adult males affects the way they are perceived by others. Ten male subjects between the ages of 25 and 35 participated in the study, along with ten other people who served as judges. Each of the ten subjects had a full head of hair at the beginning of the study. A video was created for each subject to record his pre-study appearance. Then, each subject's head was shaved completely, and a second video recording was made for each subject.

Each of the ten judges was shown the 10 original videos of the subjects with hair, as well as 2 of the 10 videos taken after shaving. Thus, each judge saw 12 videos of 10 men, with 2 of the 10 seen twice (once with hair and once without). After viewing each video, each judge was asked to complete a questionnaire about the subject in the video recording. Judges were not told the purpose of the study or that some of the viewed videos featured the same subject with different hair status (with hair vs. without). Each completed questionnaire was used to derive multiple response variables, including perceived height, strength, dominance, and attractiveness.

Let $i = 1, \dots, 10$ index the 10 subjects. Let $j = 1, \dots, 10$ index the 10 judges. Let k index hair status, where $k = 1$ indicates with hair and $k = 2$ indicates a shaved head. Let y_{ijk} be the measure of attractiveness obtained from the questionnaire completed by judge j for the video in which subject i has hair status k . Suppose

$$y_{ijk} = \mu_k + a_i + b_j + e_{ijk}, \quad (1)$$

where $a_1, \dots, a_{10} \sim N(0, \sigma_a^2)$, $b_1, \dots, b_{10} \sim N(0, \sigma_b^2)$, the e_{ijk} terms are $N(0, \sigma_e^2)$, all random effects and random errors are mutually independent, and $\mu_1, \mu_2, \sigma_a^2, \sigma_b^2$, and σ_e^2 are unknown fixed parameters.

Table 1 on page 3 shows which subjects were seen once and which were seen twice by each judge. As explained previously, when a subject was seen twice by a judge, one of the videos showed the subject with hair and the other showed the subject with a shaved head.

Table 1. Number of times each subject was seen by each judge.

Subject (i)	Judge (j)									
	1	2	3	4	5	6	7	8	9	10
1	2	2	1	1	1	1	1	1	1	1
2	2	2	1	1	1	1	1	1	1	1
3	1	1	2	2	1	1	1	1	1	1
4	1	1	2	2	1	1	1	1	1	1
5	1	1	1	1	2	2	1	1	1	1
6	1	1	1	1	2	2	1	1	1	1
7	1	1	1	1	1	1	2	2	1	1
8	1	1	1	1	1	1	2	2	1	1
9	1	1	1	1	1	1	1	1	2	2
10	1	1	1	1	1	1	1	1	2	2

Let \bar{y}_1 be the average of all the y_{ijk} values where $k = 1$. Let \bar{y}_2 be the average of all the y_{ijk} values where $k = 2$. When answering the following questions, assume that model (1) holds.

- Find the expected value of $\bar{y}_1 - \bar{y}_2$ in terms of model (1) parameters.
- Find the variance of $\bar{y}_1 - \bar{y}_2$ in terms of model (1) parameters.

Use the following partially completed sequential (type I) ANOVA table to complete the remaining parts of this problem.

Source	DF	Sum of Squares	Mean Square	Expected Mean Square
<i>Hair Status</i>	1	17.1	17.1	
<i>Subject</i>	9	115.2	12.8	$12\sigma_a^2 + 0.15\sigma_b^2 + \sigma_e^2$
<i>Judge</i>	9	71.1	7.9	$11.85\sigma_b^2 + \sigma_e^2$
<i>Error</i>	100	130.0	1.3	σ_e^2

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- Provide the value of an unbiased estimator of σ_a^2 .
- Derive the expected mean square for *Hair Status* that is missing from the ANOVA table.