STAT 510

EXAM 2

SPRING 2016

POINTS PER PROBLEM

1. 12	3. 25
2. a) 6	4. a) 6
b) 10	b) 10
c) 3	c) 10
d) 3	d) 15

1. $[\underset{\sim}{p}_1'\underset{\sim}{y}, \ldots, \underset{\sim}{p}_n'\underset{\sim}{y}]' = P'\underset{\sim}{y}$, where $P = [\underset{\sim}{p}_1, \ldots, \underset{\sim}{p}_n]$.

Because linear transformations of multivariate normal vectors are multivariate normal, we know

$$P'\underset{\sim}{y} \sim N(E(P'\underset{\sim}{y}), \mathrm{VAR}(P'\underset{\sim}{y})).$$

$$E(P'\underset{\sim}{y}) = P'E(\underset{\sim}{y}) = P'\underset{\sim}{0} = \underset{\sim}{0}.$$

$$\mathrm{VAR}(P'\underset{\sim}{y}) = P'\,\mathrm{VAR}(\underset{\sim}{y})P = P'\Sigma P.$$

By the Spectral Decomposition Theorem, we know

$$\Sigma = P\Lambda P', \text{ where } \Lambda = \mathrm{Diag}(\lambda_1, \ldots, \lambda_n).$$

Thus, $\mathrm{Var}(P'\underset{\sim}{y}) = P'\Sigma P = P'P\Lambda P'P = I\Lambda I = \Lambda$.

So, $P'\underset{\sim}{y} \sim N(\underset{\sim}{0}, \Lambda)$.

2a) WHOLE-PLOT EXPERIMENTAL UNITS ARE COOLERS.

SPLIT-PLOT EXPERIMENTAL UNITS ARE CUTS OF BEEF.

2b)

SOURCE	DF
TEMP	2
COOLER (TEMP)	9
PRES.	1
TEMP x PRES.	2
ERROR = PRES. x COOLER(TEMP)	9
C. TOTAL	23

2c) COOLER(TEMP)

2d) ERROR OR, EQUIVALENTLY, PRES. x COOLER(TEMP).

3. LET B = BLOCK, S = SOIL TREATMENT, V = VARIETY, K = KERNEL TREATMENT

SOURCE		DF	
B		3	$= 4-1$
S	WHOLE-PLOT ERROR	1	$= 2-1$
B×S		3	$= (4-1)(2-1)$
V	SPLIT-PLOT ERROR	1	$= 2-1$
S×V		1	$= (2-1)(2-1)$
B×V + B×S×V		6	$= (4-1)(2-1)+(4-1)(2-1)$
K	SPLIT-SPLIT-PLOT ERROR	1	$= (2-1)$
S×K		1	$= (2-1)(2-1)$
V×K		1	$= (2-1)(2-1)$
S×V×K		1	$= (2-1)(2-1)(2-1)$
B×K + B×S×K + B×V×K + B×S×V×K		12	$= (4-1)(1+1+1+1)$
ERROR = KERNEL (B, S, V, K)		288	$= (10-1)(4\times2\times2\times2)$
C. TOTAL		319	$= 320-1$

THIS IS AN EXAMPLE OF A SPLIT-SPLIT-PLOT EXPERIMENT.

4. a) $\bar{Y}_1 = \dfrac{100\mu_1 + 10a_1 + \cdots + 10a_{10} + 10b_1 + \cdots + 10b_{10} + \sum_{i=1}^{10}\sum_{j=1}^{10} e_{ij1}}{100}$

$= \mu_1 + \bar{a}_. + \bar{b}_. + \bar{e}_{..1}$

$\bar{Y}_2 = (20\mu_2 + 2a_1 + \cdots + 2a_{10} + 2b_1 + \cdots + 2b_{10} +$

$e_{112} + e_{122} + e_{212} + e_{222} + e_{332} + e_{342} + e_{432} + e_{442}$

$+ e_{552} + e_{562} + e_{652} + e_{662} + e_{772} + e_{782} + e_{872} + e_{882}$

$+ e_{992} + e_{9,10,2} + e_{10,9,2} + e_{10,10,2})/20$

$= \mu_2 + \bar{a}_. + \bar{b}_. +$ AVERAGE OF 20 INDEPENDENT e TERMS.
WE WILL CALL $\bar{e}_2$

$\bar{Y}_1 - \bar{Y}_2 = \mu_1 - \mu_2 + \bar{e}_{..1} - \bar{e}_2.$ $E(\bar{Y}_1 - \bar{Y}_2) = \mu_1 - \mu_2$

4 b) $\mathrm{VAR}(\bar{Y}_1 - \bar{Y}_2) = \mathrm{VAR}(\bar{\varepsilon}_{\cdot 1} - \bar{\varepsilon}_2)$

$$= \frac{\sigma_e^2}{100} + \frac{\sigma_e^2}{20} = \frac{3}{50}\sigma_e^2$$

4 d) THE SUM OF SQUARES IS $\underset{\sim}{y}'(P_2 - P_1)\underset{\sim}{y}$,

WHERE $X_1 = \underset{120\times 1}{\underset{\sim}{1}}$, $X_2 = \begin{bmatrix} \underset{100\times 1}{\underset{\sim}{1}} & \underset{100\times 1}{\underset{\sim}{0}} \\ \underset{20\times 1}{\underset{\sim}{0}} & \underset{20\times 1}{\underset{\sim}{1}} \end{bmatrix}$, $P_1 = X_1(X_1'X_1)^- X_1'$,

$P_2 = X_2(X_2'X_2)^- X_2'$, AND $\underset{\sim}{y}$ HAS ALL THE WITH HAIR OBSERVATIONS FIRST AND THE WITHOUT HAIR OBSERVATIONS FOLLOWING.

4 d) (CONTINUED)

$P_1 \underset{\sim}{y}$ IS $\bar{Y}$ (OVERALL AVERAGE) REPEATED 120 TIMES.

NOTE $\bar{Y} = \dfrac{100\,\bar{Y}_1 + 20\,\bar{Y}_2}{120}$

$P_2 \underset{\sim}{y}$ IS $\begin{bmatrix} \bar{Y}_1 \underset{100\times 1}{\underset{\sim}{1}} \\ \\ \bar{Y}_2 \underset{20\times 1}{\underset{\sim}{1}} \end{bmatrix}$.

THUS, $(P_2 - P_1)\underset{\sim}{y} = P_2 \underset{\sim}{y} - P_1 \underset{\sim}{y}$

IS $\begin{bmatrix} (\bar{Y}_1 - \bar{Y}) \underset{100\times 1}{\underset{\sim}{1}} \\ \\ (\bar{Y}_2 - \bar{Y}) \underset{20\times 1}{\underset{\sim}{1}} \end{bmatrix}$

$\underset{\sim}{y}'(P_2 - P_1)\underset{\sim}{y}$ IS JUST THE SUM OF THE SQUARES OF THESE ELEMENTS.

4 d) (CONTINUED)

$$\bar{y}_1 - \bar{y} = \bar{y}_1 - \frac{100\bar{y}_1 + 20\bar{y}_2}{120} = \frac{20\bar{y}_1 - 20\bar{y}_2}{120}$$

$$= \frac{\bar{y}_1 - \bar{y}_2}{6}$$

$$\bar{y}_2 - \bar{y} = \bar{y}_2 - \frac{100\bar{y}_1 + 20\bar{y}_2}{120} = \frac{100\bar{y}_2 - 100\bar{y}_1}{120}$$

$$= \frac{5}{6}(\bar{y}_2 - \bar{y}_1)$$

THUS, THE SUM OF SQUARES IS

$$100\frac{(\bar{y}_1 - \bar{y}_2)^2}{36} + 20 \times \frac{25}{36}(\bar{y}_1 - \bar{y}_2)^2 = \frac{50}{3}(\bar{y}_1 - \bar{y}_2)^2$$

4 d) (CONTINUED)

SS = MS IN THIS CASE, SO

$$E(MS) = \frac{50}{3} E(\bar{Y}_1 - \bar{Y}_2)^2$$

$$= \frac{50}{3} E(\mu_1 - \mu_2 + \bar{e}_{..1} - \bar{e}_2)^2$$

$$= \frac{50}{3}\left[(\mu_1 - \mu_2)^2 + E(\bar{e}_{..1} - \bar{e}_2)^2\right]$$

$$= \frac{50}{3}\left[(\mu_1 - \mu_2)^2 + VAR(\bar{e}_{..1} - \bar{e}_2)\right]$$

$$= \frac{50}{3}(\mu_1 - \mu_2)^2 + \sigma_e^2 .$$

4c) $$\frac{MS_{SUBJECT} - \frac{.15}{11.85} MS_{JUDGE} - \frac{11.70}{11.85} MS_{ERROR}}{12}$$

$$= \frac{12.8 - \frac{.15}{11.85} 7.9 - \frac{11.70}{11.85} 1.3}{12}$$

PERHAPS IT IS EASIER TO SEE AS FOLLOWS:

$$\hat{\sigma}_e^2 = MSE = 1.3$$

$$\hat{\sigma}_b^2 = \frac{MSJ - MSE}{11.85} = \frac{7.9 - 1.3}{11.85}$$

$$\hat{\sigma}_s^2 = \frac{MSS - 0.15\,\hat{\sigma}_b^2 - \sigma_e^2}{12} = \frac{12.8 - 0.15\left(\frac{7.9 - 1.3}{11.85}\right) - 1.3}{12}$$

WHICH IS THE SAME AS THE EXPRESSION ABOVE.