STAT 510 Homework 6

Due Date: 11:00 A.M., Wednesday, March 4

1. Consider the plant density example discussed in slide set 6.

   (a) For each of the tests in the ANOVA table on slide 38, provide a vector $\boldsymbol{c}$ so that a test of $H_0 : \boldsymbol{c}'\boldsymbol{\beta} = 0$ would yield the same statistic and p-value as the ANOVA test. (You can use R to help you with the computations like we did on slides 45 and 46 of slide set 6.) Label these vectors $\boldsymbol{c}_1, \boldsymbol{c}_2, \boldsymbol{c}_3$, and $\boldsymbol{c}_4$ for the linear, quadratic, cubic, and quartic tests, respectively.

   (b) Are $\boldsymbol{c}_1'\boldsymbol{\beta}$, $\boldsymbol{c}_2'\boldsymbol{\beta}$, $\boldsymbol{c}_3'\boldsymbol{\beta}$, and $\boldsymbol{c}_4'\boldsymbol{\beta}$ contrasts? Explain.

   (c) Are $\boldsymbol{c}_1'\boldsymbol{\beta}$, $\boldsymbol{c}_2'\boldsymbol{\beta}$, $\boldsymbol{c}_3'\boldsymbol{\beta}$, and $\boldsymbol{c}_4'\boldsymbol{\beta}$ orthogonal? Explain.

2. Suppose $\boldsymbol{H}$ is a symmetric matrix. Prove that $\boldsymbol{H}$ is nonnegative definite if and only if all its eigenvalues are nonnegative. (If you wish, you may use the Spectral Decomposition Theorem in your proof.)

3. Consider the model

$$y_i = \mu + x_i \epsilon_i,$$

   where for $i = 1, \ldots, n$, y_i is the response for observation i, μ is an unknown real-valued parameter, x_i is the ith known nonzero observation of an explanatory variable, $\epsilon_1, \ldots, \epsilon_n$ are independent and identically distributed as $N(0, \sigma^2)$, and $\sigma^2 > 0$ is an unknown variance component. Provide an expression for the best linear unbiased estimator of μ. Simplify your answer as much as possible.

4. Suppose

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu \\ 2\mu \end{bmatrix}, \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \right).$$

   (a) A linear unbiased estimator of μ has the form $\boldsymbol{a}'\boldsymbol{y} = a_1y_1 + a_2y_2$ for some real-valued constants a_1 and a_2. What has to be true about a_1 and a_2 in order for $\boldsymbol{a}'\boldsymbol{y} = a_1y_1 + a_2y_2$ to be unbiased?

   (b) Write a simplified expression for the variance of $\boldsymbol{a}'\boldsymbol{y} = a_1y_1 + a_2y_2$ in terms of a_1 and a_2.

   (c) Use the result of part (a) to write the answer to part (b) in terms of a single variable.

   (d) Use parts (a) through (c) and a calculus-based argument to derive the BLUE of μ.

   (e) Use the result on slide 12 of slide set 10 to determine the BLUE of μ.

5. Consider the Aitken Model with normal errors described on slide 18 of slide set 10. As usual, assume that $\boldsymbol{y}$ is $n \times 1$ and that the rank of $\boldsymbol{X}$ is r. Suppose $\boldsymbol{c}'\boldsymbol{\beta}$ is estimable. Give an expression for a 95% confidence interval for $\boldsymbol{c}'\boldsymbol{\beta}$ in terms of $\boldsymbol{y}$, $\boldsymbol{X}$, $\boldsymbol{V}$, $\boldsymbol{c}$, n, and r. [Hint: Write the expression for the confidence interval in terms of the transformed model that involves $\boldsymbol{z}$ and $\boldsymbol{W}$, and then make substitutions.]